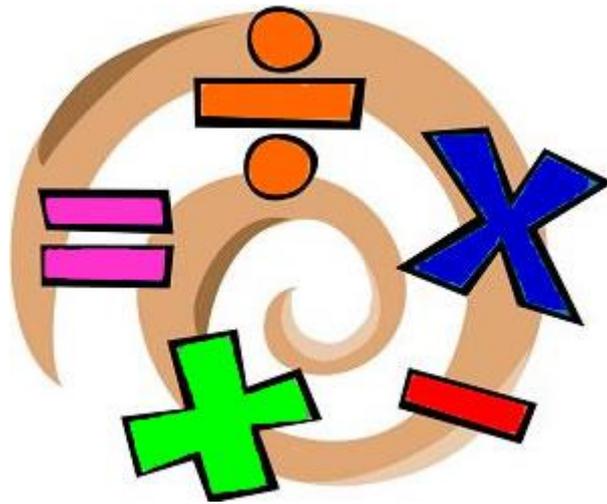




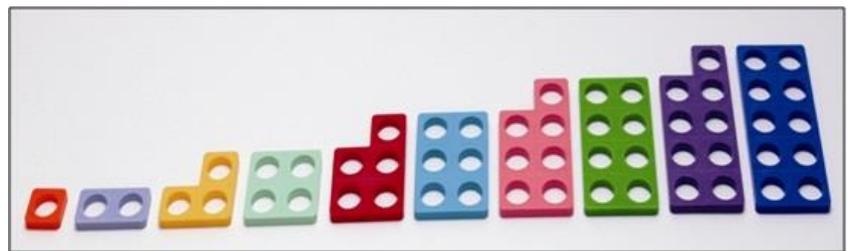
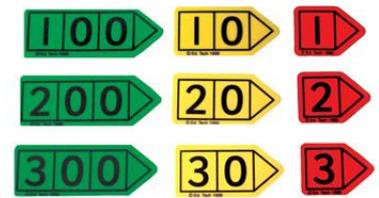
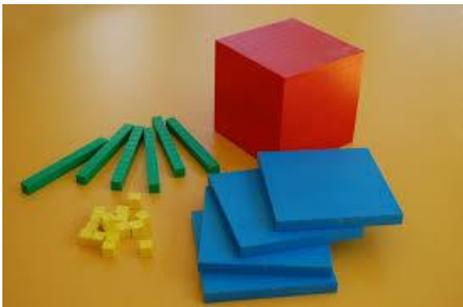
St. Andrew's C E Primary and Nursery School



Progression in the Teaching of Addition and Subtraction.

A Guide For Parents

Whilst teaching addition and subtraction we always begin with **concrete** materials to help the children develop a tangible understanding of number.



They will then move onto developing **visual** ways of recording which will ultimately develop their **abstract** methods of calculation.

Concrete – Visual – Abstract

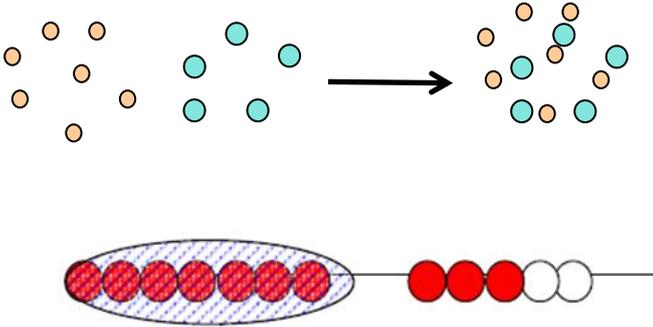
Addition

Combining two sets

Children are introduced to addition through practical materials (counters, cubes, pennies, bead strings, numicon etc.)

Two or more amounts or numbers are put together to make a total

$$7 + 5 = 12$$



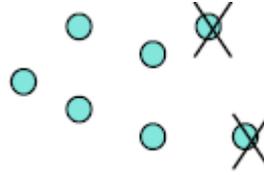
Initially the children will combine the two groups and count the total beginning at 1.

Subtraction

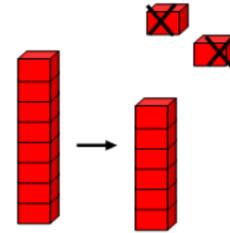
Taking away

We begin taking away using concrete materials. One quantity is taken away from another to calculate what is left.

$$7 - 2 = 5$$

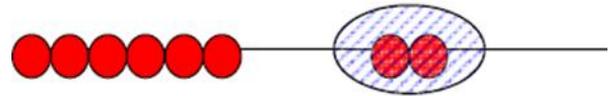


Multilink towers - to physically take away objects.



Bead strings:

$$8 - 2 = 6$$

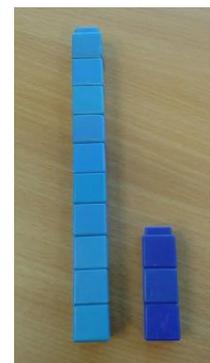
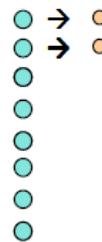


Finding the difference

The children are taught to find the difference between the two numbers

$$8 - 2 = 6$$

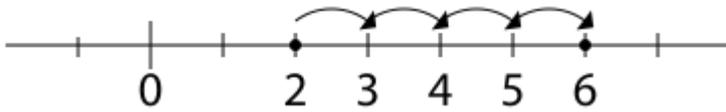
Counters:



The children will then learn to count from the starting number and add on.

e.g $2 + 4$

Begin on a number line at 2 and jump on 4

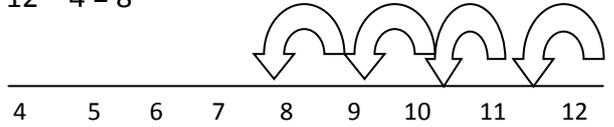


Put 2 in head and count on 4 to land on 6

The children also learn to subtract using a number line by jumping back from the starting number.

e.g

$$12 - 4 = 8$$



Counting up or 'Shop keepers' method

Using a bead string the children learn to subtract by counting on.

$$12 - 7$$



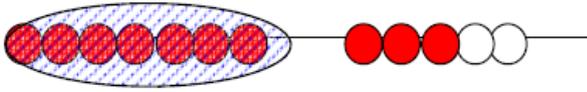
Starting from 7 on the bead string we can work out the difference by counting on. The children investigate 'How many more do we need to get to 12?'

This can become a mental strategy which we often use to find the difference between two numbers.

Bridging through 10s

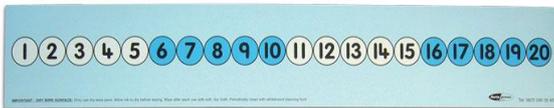
This stage encourages children to become more efficient and begin to employ known facts.

Bead string:



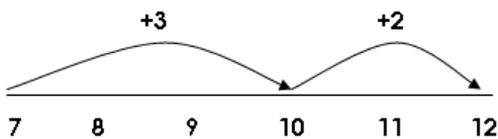
$7 + 5$ is decomposed / partitioned into $7 + 3 + 2$.
The bead string illustrates 'how many more to the next multiple of 10?' (children should identify how their number bonds are being applied) and then 'if we have used 3 of the 5 to get to 10, how many more do we need to add on? (ability to decompose/partition all numbers applied)

Number track:



Steps can be recorded on a number track alongside the bead string, prior to transition to number line.

Number line

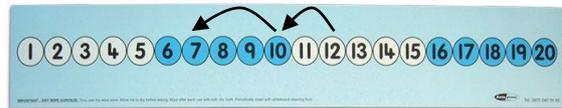


Bead string:



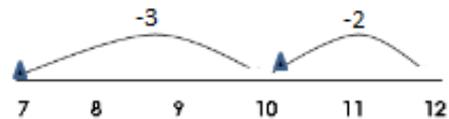
$12 - 7$ is decomposed / partitioned in $12 - 2 - 5$.
The bead string illustrates 'from 12 how many to the last/previous multiple of 10?' and then 'if we have used 2 of the 7 we need to subtract, how many more do we need to count back? (ability to decompose/partition all numbers applied)

Number Track:



Steps can be recorded on a number track alongside the bead string, prior to transition to number line.

Number Line:



Counting up or 'Shop keepers' method

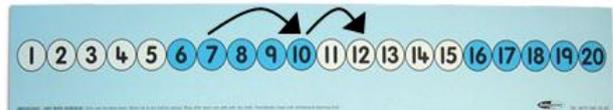
Bead string:



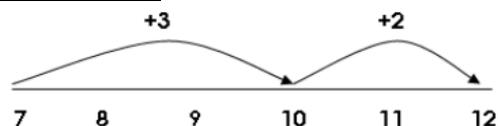
$12 - 7$ becomes $7 + 3 + 2$.

Starting from 7 on the bead string 'how many more to the next multiple of 10?' (children should recognise how their number bonds are being applied), 'how many more to get to 12?'

Number Track:



Number Line:

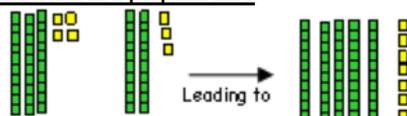


Working with larger numbers
Tens and ones + tens and ones

Partitioning

$34 + 23 = 57$

Base 10 equipment:



Children begin by creating the two sets with Base 10 equipment and then combine; ones with ones, tens with tens.

This can also be investigated using number discs/ numicon etc.

They will then move on to adding using a number line.

Number line: $34 + 23 = 57$



At this stage, children may begin to use an informal method to support, record and explain their method.

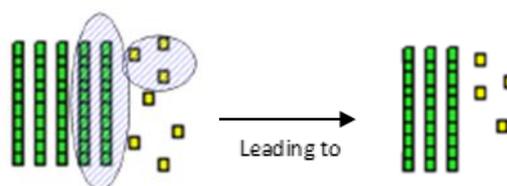
$$\begin{array}{ccccccc}
 30 & + & 4 & + & 20 & + & 3 \\
 \underbrace{\hspace{1.5cm}} & & \underbrace{\hspace{1.5cm}} & & \underbrace{\hspace{1.5cm}} & & \underbrace{\hspace{1.5cm}} \\
 50 & & & & & & 7 \\
 & & & & \underbrace{\hspace{1.5cm}} & & \\
 & & & & 57 & &
 \end{array}$$

Take away

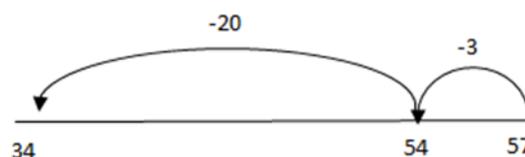
$57 - 23 = 34$

Base 10 equipment:

Children remove the lower quantity from the larger set, starting with the ones and then the tens. In preparation for formal decomposition.



Number Line:



Some children may also use an informal method to support, record and explain their method at this stage.

$$\begin{array}{ccccccc}
 (50 & + & 7) & - & (20 & + & 3) \\
 \underbrace{\hspace{1.5cm}} & & \underbrace{\hspace{1.5cm}} & & \underbrace{\hspace{1.5cm}} & & \underbrace{\hspace{1.5cm}} \\
 30 & & & & 4 & & \\
 & & & & \underbrace{\hspace{1.5cm}} & & \\
 & & & & 34 & &
 \end{array}$$

Commutativity

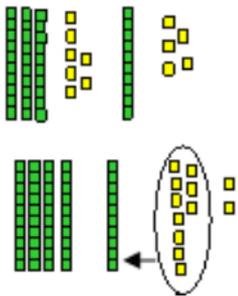
The children will learn that $3 + 4$ has the same total as $4 + 3$

Children learn that subtraction is **not commutative** i.e that the numbers in the subtraction sentence cannot be swapped around to get the same answer.

Bridging with larger numbers

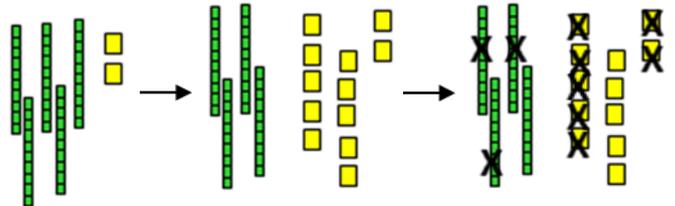
Base 10 equipment: Using the Base 10 equipment, children exchange ten ones for a single tens rod.

$$37 + 15 = 52$$



Base 10 equipment: Using the base 10 materials the children learn to exchange a whole 10 for 10 ones.

(Knock on the door – “Can the 10 come to join us?”)
 $52 - 37 = 15$



Expanded Vertical Method

Some children may benefit from consolidating their understanding of partitioning to support subtraction and addition by using the following expanded method.

Base 10 equipment:

$$67 + 24 = 91$$

67	60 + 7
+ 24	20 + 4
11	7 + 4
80	60 + 20
91	

$$60 + 7 + 20 + 4 =$$

80
11

91

Base 10 equipment:

$$91 - 67 = 24$$

80	11
90 + 1	
- 60 + 7	
20 + 4	

Many children however will be able to move straight on to the compact method of addition and subtraction. Initially they will use base 10 materials to ensure that they understand what is happening as they exchange 10s for ones and vice versa.

Compact method

Leading to

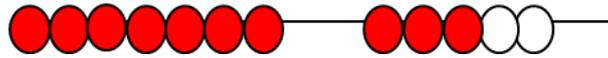
Leading to

Compact decomposition

Decimals

The children need to be confident in counting forwards and backwards in decimals – using bead strings to support.

Bead strings:



Each bead represents 0.1, each different block of colour equal to 1.0

Base 10 equipment:

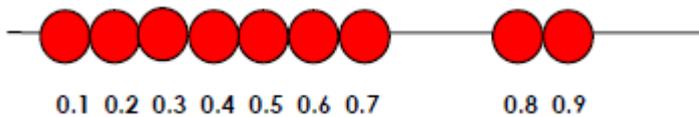


Addition of decimals

Combining two sets

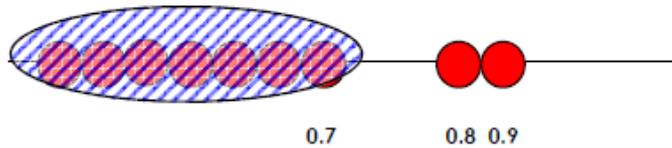
Counting both sets – starting at zero.

$$0.7 + 0.2 = 0.9$$



The children will then start counting from the first set total and count on to the end of the second set.

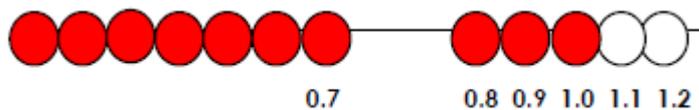
$$0.7 + 0.2 = 0.9$$



Bridging through 1.0

The children will begin adding on a bead string or a number line.

$$0.7 + 0.5 = 1.2$$



Partitioning Using partitioning the children will add the tenths and ones, exchanging 10 tenths for a one physically.

$$3.7 + 1.5 = 5.2$$

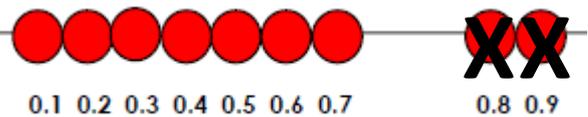


Eventually the children will build up to add decimals using the column method.

Subtraction of decimals

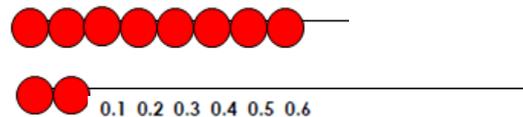
Take away model

$$0.9 - 0.2 = 0.7$$



Finding the difference (or shop keeper's model):

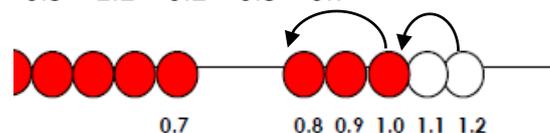
$$0.8 - 0.2 =$$



Bridging through 1.0

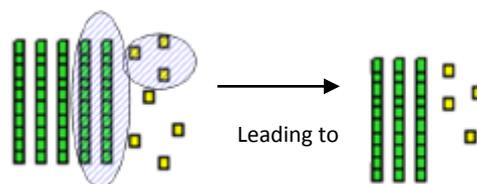
The children will begin subtracting on a bead string or a number line.

$$1.2 - 0.5 = 1.2 - 0.2 - 0.3 = 0.7$$



Partitioning Using partitioning the children will then subtract the tenths and ones.

$$5.7 - 2.3 = 3.4$$

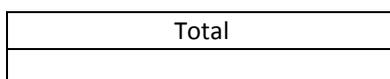


Eventually the children will build up to subtract decimals using the column method.

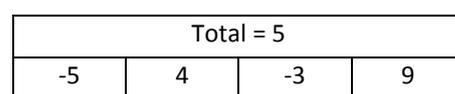
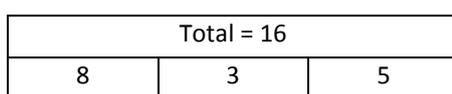
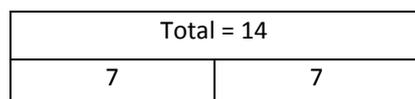
Vocabulary of Addition	Vocabulary of subtraction
add, addition, more, plus make, sum, total altogether, score, double, near double, one more, two more... ten more... one hundred more, how many more to make...? how many more is... than...? how much more is...?	subtract, take away, difference, minus, less than, leave, how many are left/left over? how many have gone? one less, two less, ten less... how many fewer is... than...? how much less is...? difference between, equals sign, is the same as?

Singapore Bar Method

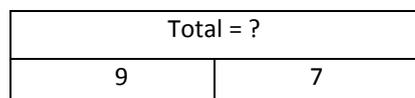
The bar method helps to consolidate the children's understanding that subtraction and addition are inverse operations. It is based on the principle that both bars are equal (Like the symbol for equals =)



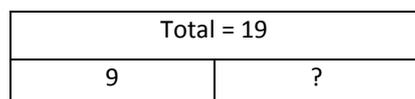
Whichever way you cut up the bottom bar, the pieces will total or equal the top bar.



Depending upon which parts of the bar they have, the children learn to calculate the missing parts. This may involve addition or subtraction.



The total is $9 + 7 = 16$



The missing part = 10 because

$$19 - 9 = 10$$

Using the different parts the children are able to develop their understanding of number sentences and number bonds.
e.g

